

QP Code : NP-18610

(3 Hours)

[Total Marks : 80

- N.B.** (1) Question no. 1 is compulsory.
 (2) Solve any three questions out of the remaining Q.no. 2 to Q. no. 6.

1. (a) Find the inverse Laplace transform of

$$\frac{S^2 + 5}{(S^2 + 4S + 13)^2}$$

- (b) If $V = 3x^2y + 6xy - y^3$, show that the function V is harmonic, find the corresponding analytic function.

- (c) Evaluate $\int_C \bar{z} dz$ where C is the upper half of the circle $r = 1$.

- (d) Prove that $f_1(x) = 1$, $f_2(x) = x$, $f_3(x) = \frac{3x^2 - 1}{2}$ are orthogonal over $(-1, 1)$.

2. (a) Evaluate $\int_0^{\infty} \frac{\cos at - \cos bt}{t} dt$

- (b) Obtain complex form of fourier series $f(x) = e^{-ax}$ for in $(-\pi, \pi)$

- (c) Using Crank-Nicholson simplified formula solve, $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$

$$u(0, t) = 0, u(4, t) = 0, u(x, 0) = \frac{x}{3}(16 - x^2)$$

Find u_{ij} for $i = 0, 1, 2, 3, 4$, and $j = 0, 1, 2$.

3. (a) Evaluate $\int_C \frac{\sin^6 z}{(z - \pi/6)^5} dz$ where C is $|z| = 1$

- (b) Find the fourier expansion for $f(x) = x - x^2 - 1 < x < 1$

- (c) Determine the solution of one dimensional heat equation, $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ under the

boundary conditions $u(0, t) = 0$ $u(\ell, t) = 0$ and $u(x, 0) = x$, ($0 < x < \ell$), ℓ being length of the rod.

4. (a) Find inverse Laplace transform by using convolution theorem,

$$f(s) = \frac{s^2}{(s^2 - a^2)^2}$$

- (b) Find the image of the region bounded by $x = 0, x = 2, y = 0, y = 2$ in the Z plane under transformation $W = (1 + i)Z$.
- (c) Find all possible Laurent's expansions of the function $f(z) = \frac{7z - 2}{z(z - 2)(z + 1)}$ about $Z = -1$.
5. (a) Solve $\frac{\partial^2 u}{\partial x^2} - 32 \frac{\partial u}{\partial t} = 0$ by Bender-Schmidt method, subject to the conditions $u(0, t) = 0, u(x, 0) = 0, u(1, t) = t$ taking $h = 0.25, 0 < x < 1$.
- (b) Obtain half range sine series for $f(x)$ when

$$f(x) = x, \quad 0 < x < \frac{\pi}{2}$$

$$= \pi - x, \quad \frac{\pi}{2} < x < \pi$$

- (c) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$ by using residues, $a > 0, b > 0$
6. (a) Find the orthogonal trajectory of the family of curves $x^3 y - xy^3 = c$.
- (b) Obtain the Fourier expansion of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $0 < x < 2\pi, f(x + 2\pi) = f(x)$
- Also deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$
- (c) Solve using Laplace transform $(D^2 - 3D + 2)y = 4e^{2t}$, with $y(0) = -3, y'(0) = 5$.